Linear Algebra 2:
Parallel programming tools for exact linear algebra

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Introduction

Back in the times, when everything was sequential
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Fortunately the great time of parallelism has come...
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Introduction:

Parallel architecture: heterogeneity

- multicore [>8 cores], ccNUMA
- network [mostly infiniband]
- GPU, separate address space
- Intel MIC
- FPGA
- ...

Main characteristics:

- complexity: memory hierarchy, number of cores
- changing hardware: Net. on Chip, Integration CPU/GPU...
Challenge

How to program heterogeneous architectures?

Criteria

▶ good performances
▶ portability across architectures
▶ abstraction for simplicity

Challenging key point: scheduling as a plugin

▶ Program: description of the parallelism
e.g. which code portions are tasks
▶ Runtime: scheduling, mapping decision

3 main programming models:

1. Parallel loop [data parallelism]
2. Fork-Join (independent tasks) [task parallelism]
3. Dependent tasks with data flow dependencies [task parallelism]
Outline

Parallel programming models
  Parallel loop model
  Fork-join model
  Data flow Tasks model
  Existing solutions

Comparison Fork-Join vs Data flow
  Overhead of task management
Parallel loop model

\[ \forall i \in [0, n[ \text{ do } f(i), \]

- where \( i \neq j \Rightarrow f(i) \) and \( f(j) \) are independent,
- i.e. result is independent of the execution order of \( f(i) \) and \( f(j) \).

Reference software: OpenMP 1.0
for (int step = 0; step < 2; ++step) {
#pragma omp parallel for
    for (int i = 0; i < count; ++i) {
        A[i] = (B[i+1] + B[i-1] + 2.0*B[i]) * 0.25;
    }
}

for (int step = 0; step < 2; ++step) {
    cilk_for (int i = 0; i < count; ++i) {
        A[i] = (B[i+1] + B[i-1] + 2.0*B[i]) * 0.25;
    }
}

for (int step = 0; step < 2; ++step) {
#pragma kaapi parallel loop
    for (int i = 0; i < count; ++i) {
        A[i] = (B[i+1] + B[i-1] + 2.0*B[i]) * 0.25;
    }
}
Fork join model

- Task based program: `spawn + sync`
- Especially suited for recursive programs
- Naive canonical example: recursive Fibonacci computation

```
# OMP

void fibonacci(long* result, long n) {
    if (n < 2)
        *result = n;
    else {
        long x, y;
        #pragma omp task
        fibonacci( &x, n-1 );
        fibonacci( &y, n-2 );
        #pragma omp taskwait
        *result = x + y;
    }
}
```
Fork join model

- Task based program: `spawn + sync`
- Especially suited for recursive programs
- Naive canonical example: recursive Fibonacci computation

**Cilk+**

```c
long fibonacci(long n) {
    if (n < 2)
        return (n);
    else {
        long x, y;
        x = cilk_spawn fibonacci(n - 1);
        y = fibonacci(n - 2);
        cilk_sync;
        return (x + y);
    }
}
```
Fork join model

- Task based program: `spawn + sync`
- Especially suited for recursive programs
- Naive canonical example: recursive Fibonacci computation

```c
void fibonacci(long* result, long n) {
    if (n<2)
        *result = n;
    else {
        long x, y;
        #pragma kaapi task
        fibonacci(&x, n-1);
        fibonacci(&y, n-2);
        #pragma kaapi sync
        *result = x + y;
    }
}
```
Data flow task model

- Task based model
- Basic definition:
  - A task is ready for execution when all its inputs variables are ready
  - A variable is ready when it was written (...)
- Old languages: ID, SISAL...
- New languages/libraries: Athapascan [96], Kaapi [06], StarSs [07], StarPU [08], Quark [10]...
Data flow graph: Cholesky factorization
SmpSS

```c
#pragma smpss task write(array)
extern void compute( double* array, int count);
#pragma smpss task read(array)
extern void print( double* array, int count);
int main() {
#pragma smpss start
    compute( array, count);
    print( array, count);  // Read after write dependency
#pragma smpss sync
#pragma smpss finish
}
```

Kaapi

```c
int main() {
#pragma kaapi parallel
{
#pragma kaapi task write(array[0..count])
    compute( array, count);
#pragma kaapi task read(array[0..count])
    print( array, count);  // Read after write dependency
}  // implicit barrier at the end of Kaapi parallel region
```
## Existing solutions

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<th>Program Model</th>
<th>Architecture</th>
<th>Target App.</th>
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<td>Fork-join</td>
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<td>Divide &amp; Conquer</td>
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<td>Parallel loop</td>
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<td>+ 3.0 [08]</td>
<td>+ Fork-join</td>
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<tr>
<td>Kaapi[06-12]</td>
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<td>Multi-CPU &amp; GPUs</td>
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</tr>
<tr>
<td>StarSs [07]</td>
<td>Flat data flow</td>
<td>multi-CPU (SMPSs)</td>
<td>LinAlg</td>
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<tr>
<td></td>
<td>Flat data flow</td>
<td>multi-CPU (SMPSs)</td>
<td>LinAlg</td>
</tr>
<tr>
<td></td>
<td>Flat data flow</td>
<td>Cell (CellSs)</td>
<td>LinAlg</td>
</tr>
<tr>
<td></td>
<td>Flat data flow</td>
<td>Grid (GridSs)</td>
<td>LinAlg</td>
</tr>
<tr>
<td>StarPU [09]</td>
<td>Flat data flow</td>
<td>multi-CPU &amp; GPUs</td>
<td>LinAlg</td>
</tr>
<tr>
<td>Quark[10]</td>
<td>Flat data flow</td>
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Outline

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  Data flow Tasks model
  Existing solutions

Comparison Fork-Join vs Data flow
  Overhead of task management
Comparison Fork-Join vs Data flow

Fork-Join: OpenMP-3.0
Data flow: Kaapi

Goal: how excessive synchronizations affect performances

By studying
- impact on performances on Cholesky/LU matrix factorization
- cost of task creation and scheduling (micro benchmark: Fibonacci)
Fork-Join vs Data flow

**Strong synchronizations in Fork-Join model:**

- if task $T_1$ depend on task $T_0$ e.g. task $T_0$ produces value for task $T_1$
- spawn $T_0$; sync; spawn $T_1$; spawn $T_2$; ...
- synchronization point at “sync”: barrier that waits for all previous spawned tasks, even if concurrency exists with some tasks after the barrier
Fork-Join vs Data flow

**Strong synchronizations in Fork-Join model:**

- if task $T_1$ depend on task $T_0$ e.g. task $T_0$ produces value for task $T_1$
- spawn T0; sync; spawn T1; spawn T2; ...
- synchronization point at “sync”: barrier that waits for all previous spawned tasks, even if concurrency exists with some tasks after the barrier

**Data Flow model:**

data flow tasks to express such dependencies

- program : creates tasks
- runtime : schedule tasks according to the real dependencies
Illustration: Cholesky factorization

```c
void Cholesky( double* A, int N, size_t NB ) {

    for (size_t k=0; k < N; k += NB) {
        clapack_dpotrf( CblasRowMajor, CblasLower, NB, &A[k*N+k], N );

        for (size_t m=k+ NB; m < N; m += NB) {

            cblas_dtrsm( CblasRowMajor, CblasLeft, CblasLower, CblasNoTrans, CblasUnit, 
                         NB, NB, 1., &A[k*N+k], N, &A[m*N+k], N );
        }

        for (size_t m=k+ NB; m < N; m += NB) {

            cblas_dsyrk( CblasRowMajor, CblasLower, CblasNoTrans, 
                                 NB, NB, -1.0, &A[m*N+k], N, 1.0, &A[m*N+m], N );

            for (size_t n=k+NB; n < m; n += NB) {

                cblas_dgemm( CblasRowMajor, CblasNoTrans, CblasTrans, 
                                 NB, NB, NB, -1.0, &A[m*N+k], N, &A[n*N+k], N, 1.0, &A[m*N+n], N );

            }
        }
    }
}
```
Illustration: Cholesky factorization

```c
void Cholesky( double* A, int N, size_t NB ) {
    #pragma omp parallel
    #pragma omp single nowait
    for (size_t k=0; k < N; k += NB)
    {
        clapack_dpotrf( CblasRowMajor, CblasLower, NB, &A[k*N+k] , N );
        for (size_t m=k+ NB; m < N; m += NB)
        {
            #pragma omp task firstprivate( k, m) shared(A)
            cblas_dtrsm ( CblasRowMajor, CblasLeft, CblasLower, CblasNoTrans, CblasUnit,
                            NB, NB, 1., &A[k*N+k] , N, &A[m*N+k] , N );
        }
        #pragma omp taskwait // Barrier: no concurrency with next tasks
    for (size_t m=k+ NB; m < N; m += NB)
    {   
        #pragma omp task firstprivate( k, m) shared(A)
        cblas_dsymm ( CblasRowMajor, CblasLower, CblasNoTrans,
                     NB, NB, −1.0, &A[m*N+k] , N, 1.0 , &A[m*N+m] , N );
        for (size_t n=k+NB; n < m; n += NB)
        {
            #pragma omp task firstprivate( k, m) shared(A)
            cblas_dgemm ( CblasRowMajor, CblasNoTrans, CblasTrans,
                          NB, NB, NB, −1.0, &A[m*N+k] , N, &A[n*N+k] , N, 1.0 , &A[m*N+n] , N );
        }
        #pragma omp taskwait // Barrier: no concurrency with tasks at iteration k+1
    }
}
```
SYNC.
Illustration: Cholesky factorization

```c
void Cholesky( double* A, int N, size_t NB ){
  #pragma kaapi parallel
  for (size_t k=0; k < N; k += NB)
  {
    #pragma kaapi task readwrite(&A[k*N+k]{ld=N; [NB][NB]})
    clapack_dpotrf( CblasRowMajor, CblasLower, NB, &A[k*N+k], N );

    for (size_t m=k+ NB; m < N; m += NB)
    {
      #pragma kaapi task read(&A[k*N+k]{ld=N; [NB][NB]}) readwrite(&A[m*N+k]{ld=N; [NB][NB]})
      cblas_dtrsm ( CblasRowMajor, CblasLeft, CblasLower, CblasNoTrans, CblasUnit, NB, NB, 1., &A[k*N+k], N, &A[m*N+k], N );
    }

    for (size_t m=k+ NB; m < N; m += NB)
    {
      #pragma kaapi task read(&A[m*N+k]{ld=N; [NB][NB]}) readwrite(&A[m*N+m]{ld=N; [NB][NB]})
      cblas_dsyrk ( CblasRowMajor, CblasLower, CblasNoTrans, NB, NB, −1.0, &A[k*N+k], N, 1.0, &A[m*N+m], N );

      for (size_t n=k+NB; n < m; n += NB)
      {
        #pragma kaapi task read(&A[m*N+k]{ld=N; [NB][NB]}, &A[n*N+k]{ld=N; [NB][NB]})
          readwrite(&A[m*N+n]{ld=N; [NB][NB]})
        cblas_dgemm ( CblasRowMajor, CblasNoTrans, CblasTrans, NB, NB, NB, −1.0, &A[m*N+k], N, &A[n*N+k], N, 1.0, &A[m*N+n], N );
      }
    }
  }

  // Implicit barrier only at the end of Kaapi parallel region
}
Benchmarks

Sparse version of the above: Kaapi vs OMP codes.
Benchmarks

Sparse version of the above: Kaapi vs OMP codes.

Also confirmed by other versions of data-flow tasks:
  - PLASMA [Dongarra & Al.]
  - SMPSs [Badia & Al.]
Challenges proper to exact linear algebra

### Slicing dimensions

- Uniform block slicing leads to unbalanced load
- Varying block sizes set statically
- Dynamically adapted block sizes (work-stealing)

### Rank deficient matrices

- Block sizes revealed during execution
Overhead of task management

Algorithm: naive recursive Fibonacci computation
Fork-join model:
  - OpenMP: gcc-4.6.2
  - Cilk+ / Intel: icc-12.1.2
  - TBB 4.0

Data flow model: Kaapi-1.0.2

AMD Opteron 4 × 12 cores
**OpenMP**

```c
void fibonacci(long* result, const long n){
    if (n<2) *result = n;
    else {
        long x,y;
        #pragma omp task
        fibonacci(&x, n-1);
        fibonacci(&y, n-2);
        #pragma omp taskwait
        *result = x + y;
    }
}
```

**Cilk +**

```c
long fibonacci(long n){
    if (n < 2) return (n);
    else {
        long x, y;
        x = cilk_spawn fibonacci(n - 1);
        y = fibonacci(n - 2);
        cilk_sync;
        return (x + y);
    }
}
```

**Intel TBB**

```c
struct FibContinuation : public tbb::task {
    long* const sum; long x, y;
    FibContinuation(long* sum_):sum(sum_){}
    tbb::task* execute() { *sum = x+y; return NULL;}
};

struct FibTask : public tbb::task {
    long n; long *sum;
    FibTask(const long n_,long*const sum_):n(n_), sum(sum_){}
    tbb::task* execute() { if (n<2){*sum = n;return NULL;}
                           else {
                               FibContinuation& c = *new(allocate_continuation());
                               FibTask& b = *new( c.allocate_child() )
                               recycle_as_child_of(c);
                               n -= 2;
                               sum = &c.x;
                               c.set_ref_count(2);
                               c.spawn( b );
                            }
    }
};
```
## Results

<table>
<thead>
<tr>
<th># cores</th>
<th>Cilk+</th>
<th>TBB-4.0</th>
<th>Kaapi</th>
<th>OpenMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.063s</td>
<td>2.356s</td>
<td>0.728s</td>
<td>2.43s</td>
</tr>
<tr>
<td>8</td>
<td>0.127s</td>
<td>0.293s</td>
<td>0.094s</td>
<td>51.06s</td>
</tr>
<tr>
<td>16</td>
<td>0.065s</td>
<td>0.146s</td>
<td>0.047s</td>
<td>104.14</td>
</tr>
<tr>
<td>32</td>
<td>0.035s</td>
<td>0.072s</td>
<td>0.024s</td>
<td>No time</td>
</tr>
<tr>
<td>48</td>
<td>0.028s</td>
<td>0.049s</td>
<td>0.017s</td>
<td>No time</td>
</tr>
</tbody>
</table>

**Sequential**

<table>
<thead>
<tr>
<th></th>
<th>Cilk+</th>
<th>TBB-4.0</th>
<th>OpenMP</th>
<th>Kaapi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0904s</td>
<td>1.063s</td>
<td>2.356s</td>
<td>2.429s</td>
<td>0.728s</td>
</tr>
</tbody>
</table>

**Slowdown (\(\frac{T_i}{\text{Sequential}}\))**

<table>
<thead>
<tr>
<th></th>
<th>Sequential</th>
<th>Cilk+</th>
<th>TBB-4.0</th>
<th>OpenMP</th>
<th>Kaapi</th>
</tr>
</thead>
<tbody>
<tr>
<td>×11.7</td>
<td>×26</td>
<td>×27</td>
<td>×8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusion

Difficult choice of the parallel programming language:

- POSIX threads: set the scheduling at programming time
- OpenMP:
  - Parallel loops
  - Fork-join Tasks
  - But still no data flow capabilities
- Cilk, TBB, Kaapi:
  - Parallel loop
  - Data flow tasks model (recursive or flat)
  - annotation, library, or proper compiler
Conclusion

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▶ Cilk, TBB, Kaapi:
  ▶ Parallel loop
  ▶ Data flow tasks model (recursive or flat)
  ▶ annotation, library, or proper compiler

Towards fully adaptive parallelism

▶ Work-stealing but in a fixed set of tasks (created at start-up time)
▶ Aim at *on-the-fly tasks creations* (extraction of parallelism from sequential code)