

# Polygonal Surface Simplifications

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# polygonal surface models

- computer vision range data captured using scanners
- scientific visualization iso-surfaces extracted from volume data
- remote sensing terrain data acquired from satellite photographs
- computer graphics & computer aided geometric design  
polygonal models from subdivision of curves parametric surfaces

# applications

- storage
- transmission
- compression
- computation involving shape information finite element analysis, collision detection, visibility testing, shape recognition
- display number of polygons vs. real time display

# characterizing algorithms

## Origin

- cartography
- geographic information system GIS
- virtual reality
- computer vision
- computer graphics
- scientific visualization
- computer aided geometric design
- finite element methods
- approximation theory
- computational geometry

# characterizing algorithms

input polygonal surface with  $n$  vertices ( typically planar triangles )

output polygonal approximating surface with  $m$  vertices ( $m << n$ ) or within a specified error tolerance

## algorithm characteristics

- Speed / Quality Tradeoff minimal error and size  $\leftrightarrow$  slow algorithm  
fast algorithm  $\leftrightarrow$  lower quality
- Refinement / Decimation coarse-to-fine top-down  
fine-to-coarse bottom-up

# algorithm characteristics

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  - fast algorithm  $\rightarrow$  lower quality
- Refinement / Decimation
  - coarse-to-fine top-down
  - fine-to-coarse bottom-up

## problem characteristics

- Topology and Geometry of Input
  - set of points,
  - samples of a height field  $z(x,y)$  on regular grid or scattered data,
  - a manifold
  - a manifold with boundary
- other Attributes of Input
  - color,
  - texture,
  - surface normal
- Domain of Output Vertices
  - subset of input vertices
  - from continuous domain

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## problem characteristics

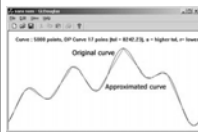
- Structure of Output Triangulation
  - regular grids
  - hierarchical subdivision, e.g. quadtree
  - general subdivision : Delaunay triangulation, data-dependent triangulation
- Approximating Elements
  - piecewise polygonal (linear)
  - quadratic, cubic, high degree polynomial
  - other basis function
- Error Metric
  - based on  $L_2$  or  $L_\infty$  norm
- Constraints on Solution
  - most accurate approximation using given number of elements
  - minimum number of elements satisfying a given error tolerance

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## curve simplification

### Douglas-Peucker'73

init: line segment PQ ( first and last point )



```

procedure LineSubd( E, P, Q )
{
  calculate S farthest point from line segment PQ
  while dist( S,PQ ) > threshold do {
    calculate left point set E'
    calculate right subset E''
    combine( LineSubd( E', P, S), LineSubd( E'', S, Q) )
  }
}
    
```

Divide-and-Conquer, worst case  $O(nm)$ , expected  $O(n \log m)$   
→ most widely used high quality algorithm

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## curve simplification

### $n$ -th point algorithm (regular sub sampling)

keep every  $k$ -th point of the input for some  $k$   
discarding the rest  
→ very fast, poor quality

### polygonal boundary reduction (decimation)

[Leu, Chen'88]  
• considers boundary arcs of 2 or 3 edges  
• computes maximum distance btw arc and chord connecting the end points  
• selects independent set of arcs whose deviation is less than some threshold and replace them by their chord  
→ improvement: bounded error,  $O(n + r^2)$  with  $r$  the number of removed vertices

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## curve simplification

### optimal approximations

search for the approximation of a given size with minimum error  
→ slow, complicated to implement

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## surface simplification

### class of surfaces

- (1) height fields and parametric surfaces
- (2) manifold surfaces
- (3) non-manifold surfaces

A manifold is a surface for which an infinitesimal neighbourhood of every point is topologically equivalent to a disc.  
In a triangulated manifold each edge belongs to two (one, in case of boundary) triangles.

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## (1) height fields and parametric surfaces

### Triangulation

- exist natural 2D parameterization of the surface
- choose topology of the triangulation by using xy-projections of the input points or by using the heights (data dependent triangulation)
- Delaunay triangulation (2 dimensional method)
  - maximizes the minimum angles of all triangulations of a given point set.
  - minimizes the occurrence of very thin *sliver* triangles.
  - empty circumcircle property.
  - algorithms: DaC  $O(m \log m)$ , incremental  $O(m^2)$ .
- data dependent triangulation
  - attempts to optimize data associated with the triangulation, e.g. the height  $z$ .
- slivers can be good when surface has high curvature in one direction

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## (1) height fields and parametric surfaces

### Methods

- 1-regular grid method : use grid of samples equally and periodically spaced in  $x$  and  $y$ .
- 2-hierarchical subdivision method : based on quadtree, k-d tree, hierarchical triangulation using DaC strategy. Recursively subdivide the surface into regions. Construct a tree-structured hierarchy.
- 3-feature method : select "feature" points in one pass and then use them for triangulation.
- 4-refinement methods : generalizations of Douglas-Peucker algorithm. Start with minimal approximation. Build up the final triangulation in multiple passes of point selection and re-triangulation.
- 5-decimation methods : begin with triangulation of all input points. Iteratively delete vertices.
- 6-optimal methods : theoretical focus on methods.

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## 1- Regular grid methods

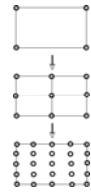
**Idea:** *regular subsampling* (equally and periodically spaced in  $x$  and  $y$ )

- simple and fast, low quality (since discarded points can be important)
- quality can be improved by low pass filtering before subsampling

general problem: *non-adaptive method*

### \* Pyramids

- hierarchical regular subsampling
- simple
- compact
- widely used in simulation area



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## 2- hierarchical subdivision methods

**Idea:**

- constructing triangulation by recursively subdividing a surface
- adaptive form of pyramids

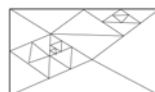
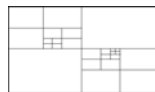
- adaptive form of subdivision
- hierarchical pattern of subdivision forms a tree (not necessarily stored explicitly in the data structure)
- Delaunay and other general triangulation algorithms don't have hierarchical topology
- fast
- simple
- facilitates MR
- yield poorer quality than more general triangulation methods.

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## 2- hierarchical subdivision methods

### \* Quadtree

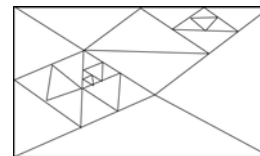
- split cells of max error
- wavelets for constructing quadtree approximations of height fields (Gross et al.'95)
- problem of cracks



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## 2- hierarchical subdivision methods

### \* Quaternary triangulation



- each triangle is recursively subdivided into 4 (nearly congruent) subtriangles until a max error tolerance is met.
- new vertices are not constrained to lie on the edges.
- cracks

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## 2- hierarchical subdivision methods

### \* Scalatos-Pavidis'92

hierarchical, recursive triangulation construction



- start with minimal triangulation
- user specifies error tolerances for each level of detail in tree
- create level  $l$  from  $l-1$  :
  - find point of highest error along each edge and in each triangle
  - keep all points with error above threshold as new vertices
  - re-triangulate each triangle using one of the 5 subdivision templates
  - repeat at same level passes of vertex selection and re-triangulation until no more candidates are found



- retain all levels of the hierarchy in data structure
- facilitating adaptive display at any detail level

$O(n \log n)$

## 3- feature methods

- make one pass over input points and rank each of them using an "importance" measure (e.g. laplacian as measure of curvature)
- select most important points
- Delaunay triangulation

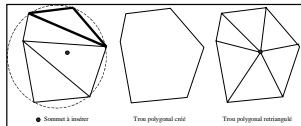
→ need good knowledge about the nature of terrains

- add data dependent triangulation, by using edge swap in order to reduce the sum of absolute errors along the edges in triangulation.

## 4- refinement methods

### \* De Floriani'83, '89

Iterative Delaunay triangulation

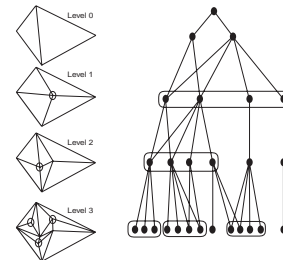


data structure: Delaunay tree

### \* De Floriani'83, '89

Delaunay tree

Structure hiérarchique introduite par Kirkpatrick [14] pour la localisation en temps logarithmique d'un triangle contenant un point dans un maillage. A gauche sont représentés les 3 niveaux de résolution. Les cercles vides indiquent les sommets insérés entre chaque niveau. A droite, les cercles solides correspondent aux triangles, et un lien rejoint deux triangles ayant une intersection non vide. Un cercle solide est rajouté pour la racine. Les rectangles arrondis correspondent aux polygones étoilés retriangulé à chaque insertion de sommet.



## 4- refinement methods

### \* Greedy insertion

- Scalatos' hierarchical structure.
- use more triangles to achieve a given error than sequential greedy insertion with Delaunay triangulation, but has the advantage of hierarchy.

### \* Garland-Heckbert'95

- two optimizations of de Floriani's algorithm

## 5- decimation methods

- start with entire input
- iteratively simplify by deleting
  - vertices
  - triangles
  - ...

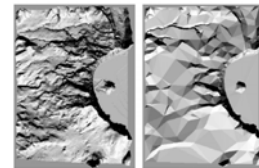


Figure 7: Original DEM data for Whistler of Creste Lake (SAGDEM vertices). Note the island in the lake.

Figure 8: Approximation using 8.0% of the input points, generated by algorithm III (7% vertices).

DEM: Digital Elevation Model

→ not so common for terrain, more for manifolds

## 6- optimal methods

## (2) Manifold Surfaces

### Methods

- 1-refinement methods : generalizations of Douglas-Peucker algorithm. Start with minimal approximation. Build up the final triangulation in multiple passes of point selection and re-triangulation.
- 2-decimation methods : begin with triangulation of all input points. Iteratively delete vertices.

## 1- refinement methods

### Problème général:

Détermination de la paramétrisation, i.e. d'un domaine de définition isomorphe à chacune des approximations, et une application de ce domaine de définition vers le maillage initial.

→ simple dans le cas des terrain data et des surfaces paramétriques. elles ont une paramétrisation 2D naturelle.

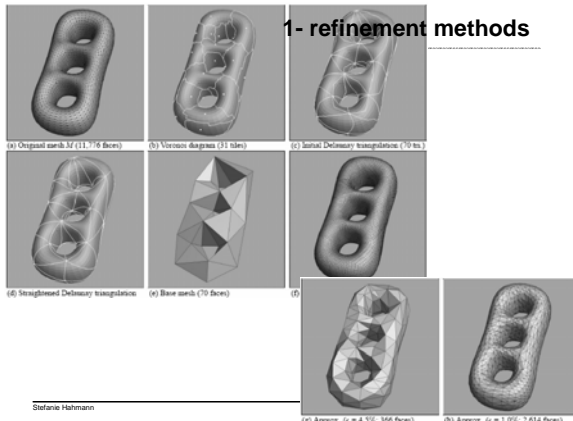
## 1- refinement methods

### \* Lounsbery, Eck et al.'95

- select few points of fine mesh, Voronoï – like
- triangulation of few points into fine mesh
- coarse triangulation
- successive refinement by 4-splitting triangles
- edge-mid-points in fine mesh
- resulting in a finer mesh than the initial one
- base mesh for wavelet analysis and synthesis

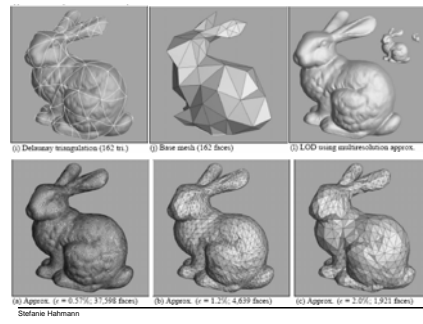
?

## 1- refinement methods



## 1- refinement methods

### \* Lounsbery, Eck et al.'95

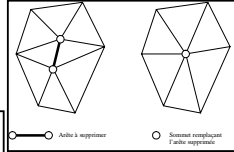
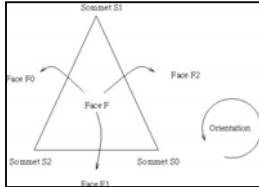


## 2- decimation methods (1)

### \* Hoppe'96 : Progressive Meshes

basic operation: edge collapse  
inverse operation: vertex split

data structure: face oriented



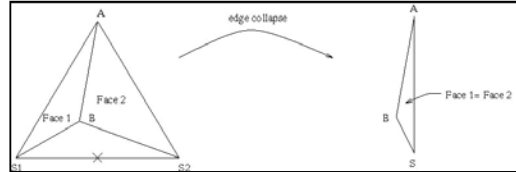
algorithm:  
iteratively select edges and collapse them.  
Save information for vertex split.

criteria for edge selection ??

### \* Hoppe'96 : Progressive Meshes

edge collapse can lead to non-manifolds configurations :

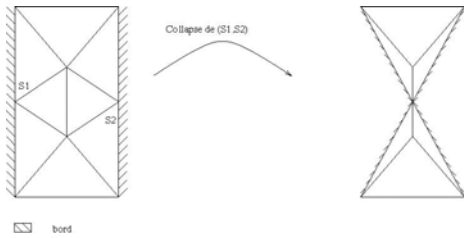
configuration 1



### \* Hoppe'96 : Progressive Meshes

edge collapse can lead to non-manifolds configurations :

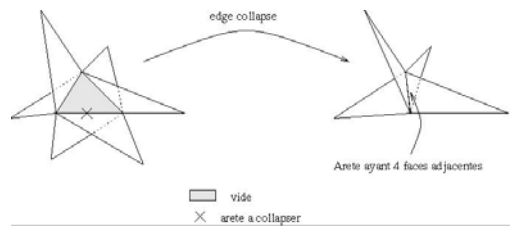
configuration 2



### \* Hoppe'96 : Progressive Meshes

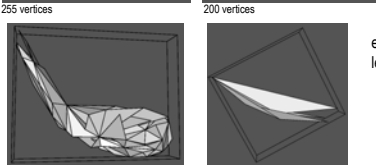
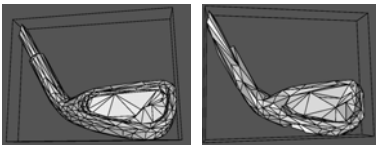
edge collapse can lead to non-manifolds configurations :

configuration 3



### \* Hoppe'96 : Progressive Meshes

Example : golf club



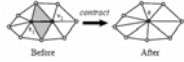
edge selection criterion:  
length of edge

### \* Hoppe'96 : Progressive Meshes



## 2- decimation methods (2)

### \* Garland Heckbert'97 : QEM



Déterminer pour chaque vertex  $v$  le  
 Quadratic Error Measure:  $\Delta v = v^T Q v$   
 Q matrice (4x4) symétrique

Posons  $Q^* = Q_1 + Q_2$  : on peut donc estimer l'erreur commise en  $v^*$  a priori

Contracter  $(v_1, v_2) \rightarrow v^*$  with  $\Delta v^* = \max(\Delta Q)$

Choix de  $v^*$ :  $v_1, v_2, (v_1 + v_2)/2$  mieux vaut  $v^*$  minimizing  $\Delta v$

#### Algorithme

1. Compute the Q matrices for all the initial vertices.
2. Select all valid pairs.
3. Compute the optimal contraction target  $\theta$  for each valid pair  $(v_1, v_2)$ . The error  $\theta^T (Q_1 + Q_2) \theta$  of this target vertex because the cost of contracting that pair.
4. Place all the pairs in a heap keyed on cost with the minimum cost pair at the top.
5. Iteratively remove the pair  $(v_1, v_2)$  of least cost from the heap, contract this pair, and update the costs of all valid pairs involving  $v_1$ .

### \* Garland Heckbert : QEM



Figure 3. Original bunny model with 8841 triangles. Rendered using the default pair in its approximation mode.



Figure 8. An approximation using only 1,000 triangles (generated in 17 seconds).



Figure 19. An approximation using only 100 triangles (generated in 17 seconds).



Figure 4. A sequence of approximations generated using our algorithm. The original model on the left has 5,824 faces. The approximations to the right have 194, 132, 248, and 64 faces respectively. Note that features such as horns and hooves continue to exist through easy simplification. Only at extremely low levels of detail do they begin to disappear.