LYAPUNOV STABILITY OF A SINGULARLY PERTURBED SYSTEM OF TWO CONSERVATION LAWS
– talk –

YING TANG, CHRISTOPHE PRIEUR, AND ANTOINE GIRARD

The singular perturbation techniques occurred at the beginning of the 20th century (see [1]). The development of this method led to the efficient use in various fields in mathematical physics and engineering, for instance, fluid mechanics, fluid dynamics, elasticity, quantum mechanics, chemical-reactor, aerodynamics etc. (see [2] for a survey). Lyapunov methods are usually used for stability analysis of dynamical systems. This is also true for those dynamics with a small perturbation parameter. In the work of [3], a quadratic-type Lyapunov function has been investigated for a singularly perturbed finite-dimensional nonlinear system.

In our work, systems modelled by singularly perturbed partial differential equations (PDEs) are considered. More precisely a class of systems of two conservation laws with a small perturbation parameter ϵ is investigated. By setting $\epsilon = 0$ the singularly perturbed system is divided as two subsystems: the reduced and boundary-layer systems. Each of the two systems has a Lyapunov function and is asymptotically stable under suitable boundary conditions. For a sufficiently small perturbation parameter, the stability of the singularly perturbed system of conservation laws can be obtained by a Lyapunov function which is given by a convex combination of the Lyapunov function of the reduced and boundary-layer systems.

The work is organized as 4 parts. The first part introduces the singularly perturbed system of conservation laws. The second part presents the stability of the reduced and boundary-layer systems. The third part shows the stability for the overall singularly perturbed system. In the last part an illustrative example is provided to show the main result.

References

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(GIPSA) UJF-Grenoble 1/CNRS, Grenoble Image Parole Signal Automatique (GIPSA-lab), UMR 5216, B.P. 46, F-38402, St Martin d'Hères, France.

(LJK) Laboratoire Jean Kuntzmann, Université de Grenoble, B.P. 53, 38041 Grenoble, France.

E-mail address: ying.tang@gipsa-lab.fr
E-mail address: christophe.prieur@gipsa-lab.fr
E-mail address: antoine.girard@imag.fr