OPTIMAL CONTROL OF A NONHOLONOMIC INTEGRATOR APPLICATION TO MICRO-SWIMMERS
– talk –

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This talk is devoted to the study of the time optimal controllability of an $(n+1)$-dimensional nonholonomic differential system with constraint on the state variables. The problem I consider arises from the motility problem in a Stokes fluid or more precisely, the swimming problem for microorganisms. This problem has been motivated by the work [6], where controllability of axi-symmetrical micro-swimmers is studied. In this talk, I will give explicit time optimal control for a problem arising from this work. The result given in this talk can be found in [5].

Nonholonomic systems have been intensively studied in numerous works and I only refer to Bloch [3] where a comprehensive survey is given in connection with control theory. Minimum time control problems for nonholonomic systems have also been considered in the literature and explicit optimal solutions have been computed when no constraint is imposed on the state variables. The case $n = 2$, i.e. when only two controls are considered, is studied in Bloch [3]. The n-dimensional control case we consider is a generalization of the Brockett integrator. The generalization that I consider is different from the one originally given in Brockett [4] but corresponds to the $(2n + 1)$-dimensional Heisenberg systems studied in Beals, Gaveau and Greiner [2] and Agrachev, Barilari and Boscain [1] in the framework of sub-Riemannian geometry. The minimal time problem for the nonholonomic system, can be interpreted as a sub-Riemannian geodesic problem. A study of the geodesics of the sub-Riemannian manifold induced by the Heisenberg group can be found in Beals, Gaveau and Greiner [2], see also Prieur and Trélat [7] or Agrachev, Barilari and Boscain [1]. All these studies are performed without any state constraint. As far as I know, there is no explicit optimal solutions when constraints are imposed on the state variables.

References

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