Dynamic modeling of clarifier-thickeners for the control of wastewater treatment plants: a critical analysis

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***Abstract*—Sedimentation is central activated sludge process, and its performance has a major impact on that of the whole wastewater treatment process. Nevertheless, there is still no satisfying model for secondary settling tanks. This paper explores the reasons why the existing one dimentional models are not relevant, from the lack of knowledge on the physical phenomena to the difficulties to solve the partial differential equations. Finally, the most important modeling challenges are presented, highlighting scientific advances that have to be done.**

Keywords*—*Wastewater treatment; clarification; model; Partial differential equations (PDE)

I. INTRODUCTION

The activated sludge process (ASP), found in most wastewater treatment plants, consists basically of a biological reactor followed by a thickener/clarifier which separates the treated wastewater from the biological sludge mass. The solid/liquid separation phase is traditionally achieved by gravity sedimentation in secondary settling tanks (SST). The SST combines the functions of:

- a thickener to produce a continuous underflow of thickened sludge for return to the bioreactor,

- a clarifier to produce a clarified effluent, which is disposed in rivers and

- a storage tank to store sludge during peak flows [1].

The behavior of the settling tanks is influenced by the design features and its operation, and sedimentation is one of the most important processes that determine the performance of the ASP. However, the biological reactor has received most attention in the literature, both for model and control design. The SST models are not yet satisfying for system design, operation optimization purposes as well as control and monitoring purposes. Recent studies have focused on the control of settlers and proposed to stabilize thickener operation: in [2] with an application on mineral processing industry and in [3]. In the last reference, the authors coupled the biological reactor and the settling tank models, leading to an ODE-PDE model. The control also aimed to regulate the sludge blanket level.

For such purposes, a relevant and efficient model is needed. First the model should be able to predict both effluent and underflow concentrations during transient operating conditions, which corresponds to clarification and thickening processes. The second main function is to approximate the concentration profile and sludge blanket level during unsteady-state operating condition. Moreover the model should be able to integrate with available bioreactor models to provide an overall secondary treatment simulation for system design and operation optimization purposes [4] and which should be used as a basis for advanced control design.

In current engineering practice, 1-D dynamic models are used and represent yet the best compromise between complexity and the representativeness of the phenomena. They are based on the flux theory: it is assumed that in clarifiers, the profiles of horizontal velocities are uniform and that horizontal gradients in concentration are negligible. Consequently only the processes in the vertical dimensions are modeled. The other general assumptions are: the constituents of the suspension are incompressible; the suspension is completely flocculated before the sedimentation begins; and that there is no mass transfer between the sediments.

The paper is organized as follows. In section II, the clarifier-thickener is described shortly together with measurement techniques to characterize the settleability of the feed effluent. Section III, focuses on the models based on mass balance. Those are based on mass and momentum balances are described in section IV.

II. CLARIFIER-THICKENERS

In its conventional form (Fig1.) activated sludge process is based on the aeration of wastewaters with mixed bacterial cultures that carry out the biological conversion of the contaminants and form the flocculated biomass known as flocs. The treated water is separated from the biomass in the settler or secondary clarifier. Part of the biomass is then wasted, and the remainder is returned to the system. The reason is to maintain a concentration of activated sludge in the aeration tank sufficient for the treatment. The excess sludge is wasted and treated for discharge or post-treatment as incineration.

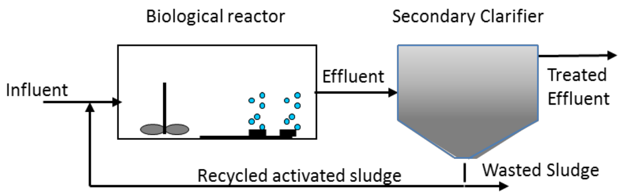

Fig. 1. Basic concept of the activated sludge process

To ensure the effective operation of the process, the secondary clarifier must fulfill two main functions: clarification and thickening. These phenomena are a consequence of the flocculation that takes place in the biological reactor. Suspended particles settle in different regimes which are shown in Fig.2. The limit between hindered settling and compression is called the sludge blanket and can be easily measured.

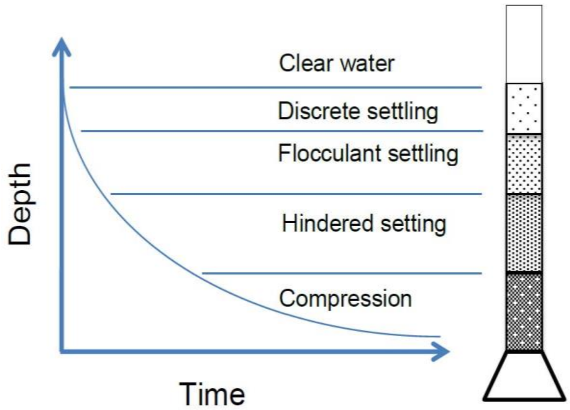

Fig. 2. Settling regions for activated sludge [5]

The settleability of the sludge is characterized by some parameters. These are influenced by the characteristics of the effluent and the hydrodynamics in the settler. The most commonly used parameter to quantify settleability is the Sludge Volume Index, SVI [1] which is based on batch tests to measure the volume of sludge occupied after a fixed period of settlement. The validity of this test is highly controversial as an indicator of settleability because the SVI is a nonspecific overall index, that is to say it does not measure a phenomenon in particular. The SVI value depends on many factors such as floc structure, diameter, and height of the settling column, agitation and in particular it can take different values for the same concentration of initial solid. To overcome these difficulties, a DSVI index (the concentration is in a range for which SVI is independent from the concentration) and a SSVI index (an agitator tends to reproduce the agitation in real settlers) can also be used.

The second parameter is the velocity of the liquid/solid interface during the zone settling stage. In order to simplify the problem, Kynch [6] made the assumption that the hindered settling velocity is uniquely determined by the local solid concentration. The most commonly used expression is the Vesilind function for which numerous experimental validations have been carried out. It corresponds to the hindered settling zone, where the velocity decreases as the concentration increases. The Velisind function is (m/h):

$$V_S(X) = V_0\, e^{-rh.X} \tag{1}$$

where V_0 is a constant [m/d] and rh is the hindered zone settling parameter [m^3/g].

To extend the flux approach to low concentrations, a double exponential settling velocity model has been proposed by Takács et al.[7] taking account of three settling fractions:

$$V_S(X) = \max\left[0, \min\left\{V_{S,max}, V_0\left(e^{-r_h(X-X_{min})} - e^{-r_P(X-X_{min})}\right)\right\}\right] \tag{2}$$

where $V_{s,max}$ is the maximal settling velocity [$m.d^{-1}$], r_P is the flocculant one settling parameter and X_{min} is the minimum concentration below which non-settleable particles are present.

The parameters V_0, r_h and r_P are usually determined by empirical equations using sludge settleability tests such as SVI. The drawbacks are that they have to be adjusted with off-line batch tests, which is incompatible with unsteady-states of operation and on-line control.

III. Model Based on Mass Conservation

A. *Tackacs'model of sedimentation*

1) Model and physical hypothesis: This model is based on the solid flux theory developed initially by Kynch [6]. The main assumptions are that the suspension is completely flocculated before the sedimentation begins and that there is no mass transfer between the solid and the fluid during sedimentation.

The settler is idealized as a continuous flow cylindrical reactor. The total sludge flux consists of the hydraulic flux ($V_H X$), and the settling flux ($V_S(X).X$). The simplest form of differential conservation equation describing this process is [1]:

$$-\frac{\partial X}{\partial t} = V_H \frac{\partial X}{\partial z} + \frac{\partial V_S(X)\,X}{\partial z} \tag{3}$$

The settling process is thus exclusively defined by this continuity equation regardless of the actual forces affecting the particles.

The hydraulic velocity V_H only depends on the flows and the surface of the layer. Equation (3) has thus two unknowns (X and V_S). Hence another equation is needed to obtain a solution. The most often used equation is the double exponential settling velocity model (Eq. 2), though other empirical expressions can be also used [4].

2) Application to settling tanks: This equation to be solved is a first order non-linear hyperbolic PDE. Equation (3) is made operational in computer programs by splitting the tank into ten horizontal layers (Fig 3.) and by discretizing equation (3) on these layers [7].

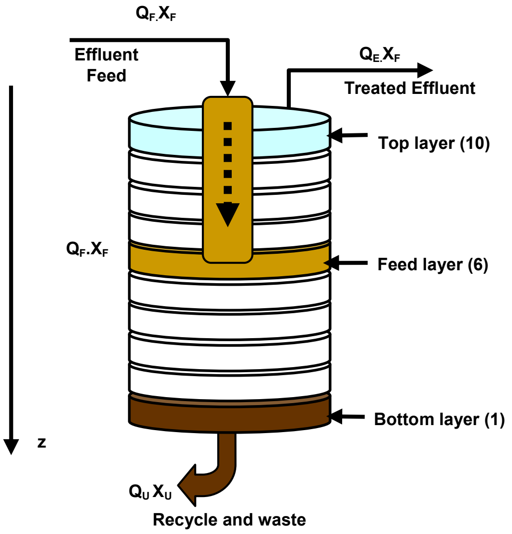

Fig. 3. Decomposition in Layers in the Takács'model

Additionally to the discontinuities caused by the discretization of the equation, some restrictions on the flux have been added. Indeed, a load increase according to equation (3) will cause a shock wave to propagate. The nature of equation (3) is such that the fronts of these waves remain sharp and mathematically discontinuous, and are not damped with time. The flux reflection of shock waves at the top and the bottom boundaries induce a complex wave pattern that finally lead to numerical instability. This is the reason why in the Takàcs'model [7], an empirical expression limits the flux from one layer to another, which prevents the mass flux into a layer to exceed the flux the layer is capable to transfer [1].

The equations of the Takàcs' model are used by the Benchmark BSM1 and can be found in [8]. For the sludge phase, the equations are different on each layer: the feed layer, the intermediate bellow the feed layer, the intermediate above the feed layer. The boundary conditions are not explicitly written and are included in the expression of the top and bottom layers. They express the fact that the flux above (respectively bellow) are null. The model is initialized at steady state values.

3) Discussion: The final model is finally not so simple to implement and numerous drawbacks have been reported in literature. One major drawback is its inability to emphasize concentration gradients for steady-states [9]. Indeed the steady-state ($\frac{\partial X}{\partial t}=0$) for settlers with constant sections leads to: $\frac{\partial X}{\partial z}=0$, which means that there is no spatial profile in steady-state, in major contradiction with the experimental evidence. An explanation for the success of the Takács' model during normal operating conditions is the following [10]: the settler is operating in continuous sedimentation, and the concentration is normally non-decreasing with depth, and the Takács method often works satisfactorily. However during extreme events, such as storm weather, the concentration distribution may be decreasing with depth, then the Takács method fails.

Several other weaknesses have also been reported in [10]: the simulations produced by the Takács method are qualitatively different for different number of layers, and increasing the number of the layers deteriorates the model performance [1]. Another interesting remark in [10] is that in a consistent modeling methodology, the numerical method for simulation should be closely related to the model equation. In particular parameters introduced directly into the numerical method mean that one imposes assumptions on the solution which may be unphysical. This is however what is done in the Takács model by the threshold concentration.

All these drawbacks have lead to look for a more representative model.

B. Extension with diffusivity

1) Model and physical hypothesis: A way to treat the propagating shock wave is to introduce an eddy diffusivity term, which leads to equation (4):

$$-\frac{\partial X}{\partial t}=V_H\frac{\partial X}{\partial z}+\frac{\partial V_S(X)X}{\partial z}-D\frac{\partial^2 X}{\partial z^2} \qquad (4)$$

Owing to the diffusion term, the gradient of a shock wave front is decreased while propagating and the numerical procedure becomes stable. In addition this convective-diffusive equation makes the final solution independent of the initial conditions. This term also helps to distinguish effects of sludge settleability (through $V_S(X)$) and hydrodynamics considered via the pseudo-diffusivity D. In the applications, the parameter D includes many effects, especially if it is used to fit the data, and is therefore called the pseudo-diffusivity coefficient. Because the diffusion depends strongly on the concentration gradient, the number of layers becomes an important value in this approach. It is also proved that an elevated sludge blanket can be obtained in a steady-state situation, where in the previous model, without a diffusive term only the bottom layer is occupied [1]. This is a needed property of the model to deal with sludge storage or to face variations in the biological reactor.

The parameter D also influences the results on the concentration profile. The easiest way is to choose D as a constant, but thus only the molecular diffusion is taken into account, and the dispersion effect due to the hydraulic disturbance is not characterized. Some empirical expressions have been proposed to improve the model that can be found in [4], but this increase the complexity of the model and add several parameters to be identified.

2) Application to settling tanks: The convection-diffusion model is a parabolic equation, which is easier to solve numerically. David and co-authors [11] proposed the Method of Lines (MOL) strategy for this problem. This method is a straightforward two-step procedure, where the PDEs are first discretized in space, and then integrated in time. The resolution has been processed in two steps: firstly in batch mode and secondly in the real case.

The settler is divided into two zones I and II (Fig.4), corresponding to the zone between the clear outlet (z_0) and the feed level (z_f), and to the zone between the feed level and the sludge outlet (z_L), respectively.

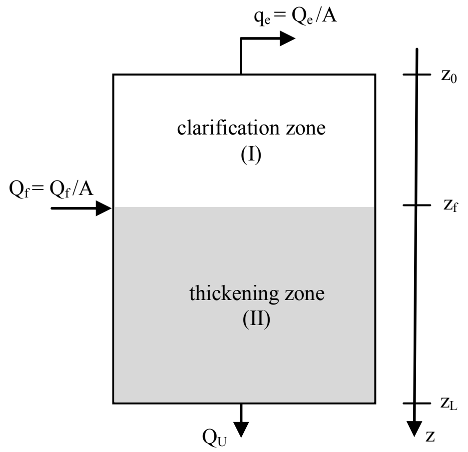

Fig. 4. Schematic overview of an ideal 1D SST divided into two zones, with A corresponding to the settler area and the volumetric flow rates corresponding to the feed (Qf), the clear water outlet (Qe) and the sludge outlet (Qu) [12]

The model then becomes:

$$
\begin{aligned}
-\frac{\partial X_I}{\partial t} &= q_e \frac{\partial X_I}{\partial z} + \frac{\partial V_{S,I}(X_I)\, X_I}{\partial z} - D_I \frac{\partial^2 X_I}{\partial z^2} \\
-\frac{\partial X_{II}}{\partial t} &= -q_u \frac{\partial X_I}{\partial z} + \frac{\partial V_{S,II}(X_{II})\, X_{II}}{\partial z} - D_{II} \frac{\partial^2 X_{II}}{\partial z^2}
\end{aligned} \tag{5}
$$

The boundary conditions express the fact that the material flows at the system boundaries are equal to zero in batch mode. The boundary conditions are:

$$
\begin{cases}
\text{At } z = z_0 \quad V_{S,I} X_I - D_I \dfrac{\partial X_I}{\partial z} = 0 \\
\text{At } z = z_f \quad X_I = X_{II} = \dfrac{q_f C_f}{q_u + q_e} \\
\text{At } z = z_L \quad V_{S,II} X_{II} - D_{II} \dfrac{\partial X_{II}}{\partial z} = 0
\end{cases} \tag{6}
$$

The initial conditions are given for an empty settler:

$$
X_I(t = 0, z_I) = 0 \text{ and } X_{II}(t = 0, z_{II}) = 0 \tag{7}
$$

The system of equations (5) has now three unknowns for each zone (X, V_S and D), and the double exponential settling velocity model (Eq. 2) is used for Vs. The simulations have been processed with the same parameter values as the Takács' model parameters, and give more realistic results: the model of Takács predicts smaller concentration values X_U at the sludge outlet. The sludge concentration X_U is also varying more in the Takács' model [12].

3) Discussion: The representativeness of the behavior of this model is limited because the compression phase of the sludge is not taken into account. Figure 5 shows the new representation of the settler. This representation is far more realistic as it shows the sludge blanket level, which is important to manage the storage capacity of the settler.

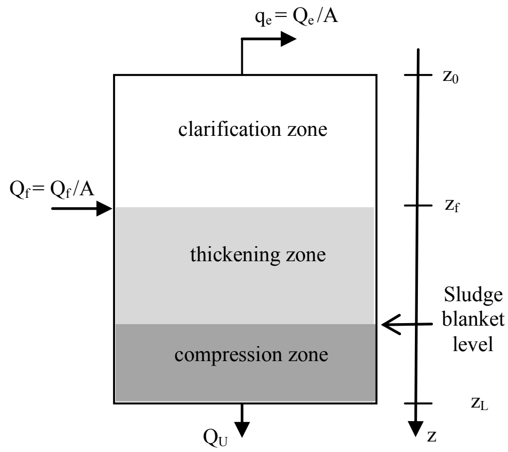

Fig. 5. Schematic overview of an ideal 1D SST with compression zone

This adds however important complexity: a term of compression must be added, and the double exponential settling velocity model is no more valid in this zone. The numerical resolution has also to face the fact that the sludge blanket level which marks a physical discontinuity is varying.

To overcome these difficulties, the equation of the momentum conservation is used.

IV. Model based on mass and momentum conservation

The momentum conservation law has been introduced to better describe the phenomena, and more especially the compression zone. These equations are based on a widely accepted mathematical theory on sedimentation-consolidation process of flocculated suspensions. Here we consider the general assumption that the solid particles are small with respect to sedimentation vessel and have the same density.

A. The momentum equations

The model is based on two-phase flow model, with mass and momentum equations for each phase: liquid (L) and sludge (S). The equations are written in batch configuration.

The mass balance equation for the liquid is :

$$-\frac{\partial(1-\varepsilon)}{\partial t}=\frac{\partial((1-\varepsilon)V_L)}{\partial z} \tag{8}$$

The mass balance equation for the sludge is :

$$-\frac{\partial\varepsilon}{\partial t}=\frac{\partial(\varepsilon V_S)}{\partial z} \tag{9}$$

The forces acting on the solids and liquid are gravity, liquid pressure and the solid-liquid interactions (Figure 6).

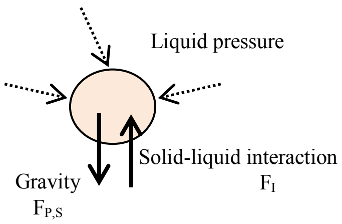

Fig. 6. Forces acting on an ideal floc structure

The liquid momentum equation is:

$$-\rho_L\frac{\partial((1-\varepsilon)V_L)}{\partial t}=\rho_L\frac{\partial((1-\varepsilon)V_L^2)}{\partial z}+F_{G,L}+F_{P,L}+F_I \tag{10}$$

The solid momentum equation is:

$$-\rho_S\frac{\partial(\varepsilon V_S)}{\partial t}=\rho_S\frac{\partial(\varepsilon V_S^2)}{\partial z}+F_{G,S}+F_{P,S}-F_I \tag{11}$$

where ε is the solid volumetric fraction; ρ_L and ρ_S are the liquid and solid densities; V_L and V_S are the liquid and solid velocities; $F_{G,L}$ and $F_{G,S}$ are the gravity forces; $F_{P,L}$ and $F_{P,S}$ are the stress forces, and F_I is the solid-liquid interaction force or drag force. $q=\varepsilon V_S+(1-\varepsilon)V_L$ is the volume average velocity of the suspension which can be controlled externally.

The expression of the gravity forces is direct:

$$\begin{cases} F_{G,S}=\varepsilon\,\rho_S\,g \\ F_{G,L}=(1-\varepsilon)\rho_L\,g \end{cases} \tag{12}$$

The liquid ($F_{P,L}$) and solid ($F_{P,S}$) pressures represents the resistance to the fluid compression, and their expression are [13][14]:

$$\begin{cases} F_{P,S}=\dfrac{\partial p_S}{\partial z}=\dfrac{\partial(\varepsilon p+\sigma_e)}{\partial z} \\ F_{P,L}=\dfrac{\partial p_L}{\partial z}=(1-\varepsilon)\dfrac{\partial p}{\partial z} \end{cases} \tag{13}$$

where p_S is the solid partial pressure, p is the pore pressure (i.e. the pressure supported by the liquid filling the interstices or pores), $\sigma_e(\varepsilon)$ is the effective solid stress function (i.e. the stress supported by the solid skeleton, which is equal to zero in the hindered zone and >0 in the compression zone), and p_L is the liquid partial pressure. The effective solid stress $\sigma_e(\varepsilon)$ is an increasing function of the concentration above a critical concentration Xc, above which the particles are in constant contact, and equal to zero below [15].

The solid-liquid interaction force, also called the drag force, is proportional to the liquid-solid relative velocity at sufficiently low Reynolds number:

$$F_I=r(V_L-V_S) \tag{14}$$

Therefore, calculating the hydrodynamic drag force is equivalent to calculating the coefficient of resistance for the relative solid-liquid flow r. Several methods have been developed [4]:

- The hindered settling factor approach, which is based on the Stokes drag coefficient with a correcting factor to take into account the interactions between the particles,
- The internal flow approach, for which the flow is considered as a flow in a porous medium and using the Darcy's law. The coefficient is determined by an empirical function based on the Velisind equation.

Both lead to similar results in practice.

B. *Application to secondary settlers*

Taking into account the momentum balances, a system of four equations (8-11) is obtained. Several functions or varying parameters have been added: $\sigma_e(\varepsilon)$, r, as well as the pore pressure p. A difficulty is that these functions or parameters are discontinuous functions of the depth variable. To solve these equations, continuous solutions are however expected, though discontinuities linked to the three main zones are the real physical phenomena. The direct resolution is so far an open challenge.

However equations (10) and (11) have been used to estimate the settling velocity in the compression zone for batch sedimentation at the equilibrium [14]. The expression of the settling velocity has thus two formulations, separated by a critical concentration X_C: for concentrations greater than X_C, the settling velocity is reduced by a compression effect when the concentration increases with depth [15]:

$$\begin{cases} V_{hS} \quad \text{for} \quad 0\le X\le X_C \\ V_S=V_{hS}\left(1-\dfrac{1}{(\rho_S-\rho_L)g\varepsilon}\dfrac{\partial\sigma_e(\varepsilon)}{\partial\varepsilon}\dfrac{\partial\varepsilon}{\partial z}\right) \text{for} \quad X>X_C \end{cases} \tag{15}$$

where $V_{hS}(\varepsilon)$ is the hindered solid velocity which can be taken from the Velisind expression.

Inserting equation (15) into the equation of mass balance (9), the expression (9) becomes in the compression zone:

$$-\frac{\partial \varepsilon}{\partial t}=\frac{\partial Vh_S\,\varepsilon}{\partial z}-\frac{\partial}{\partial z}\left(\frac{V_{hS}}{(\rho_S-\rho_L)g}\frac{\partial \sigma_e(\varepsilon)}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial z}\right) \quad (16)$$

We remark that for $X<X_C$, the model is a first-order hyperbolic equation, and for $X>X_C$, the model is a second order parabolic equation.

It is well known from the theory of nonlinear first-order laws that solutions of equation (16) may contain discontinuities, such as rising or falling jumps in the clarification and the thickening zones. These discontinuities are however a desirable property since this sharp interface is indeed observed in reality. Thus it is not straightforward to use any finite difference approximation for the derivatives, and Bürger et al. [16] proposed a method-of-lines formulation.

This model has not been validated experimentally, but the functions for the Kynch batch density function (V_{hS}), the effective solids stress σ_e, and the critical concentrations X_C have been identified experimentally [17].

V. CONCLUSION

In this paper, the one-dimensional models of secondary settling tanks have been presented. Three models of increasing accuracy on the physical phenomena have been proposed: the first one is a convection model, the second one a diffusion-convection model and the last one includes a compression zone. The drawback is that the following functions or parameters that are added to each model have to be defined: V_S, D, σ_e, and X_C. The usual method used to determine these functions is based on experimentation on batch systems, which greatly reduces the validity domain of the models. This is more particularly true for the hindered velocity function V_S, whose representation is not satisfactory as its domain of validity is very small [4].

Another problem that arises in the modelling of such kind of systems is the complexity to couple two systems described by PDE's (the clarification zone and the thickening zone) with a mobile interface (hindered/compression interface). The second difficulty is that the nature of these PDE's changes from a hyperbolic to a parabolic nature/behavior. Recently some work have been performed on the analysis of the stability and control properties of two hyperbolic PDE's coupled via a mobile interface described by an ODE e.g. for an extrusion process [18] or a diesel motor [19].

The advantages of such techniques are, to take into account the intrinsic properties of the system, and to describe the involved phenomena. But the complexities to analyze these classes of systems, to identify the parameters on the experimental setup, to tune the controller and to implement it on simulations are such relevant challenges that need to be highlighted.

Several approaches are possible, first with some simplifications on the model, at least on the diffusion term that could evaluate from a constant function to a smooth function (see [4]). The second challenge would be to find and to define an adaptive controller for such kind of problems defining exactly the control objectives.

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