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# Erratum de "A triangular central limit theorem under a new weak dependence condition", 17 septembre 2001

## 1 Résultats principaux

**Theorem 1** *Assume that the $E$-valued sequence satisfies the 's'-weak dependence condition and the triangular array $(X_{n,k})_{1\le k\le k_n}$ defined as before satisfies assumption (6), then if*

$$(k_nM_n + k_n^{\frac{2}{3}})M_n\delta_n \to 0, \quad k_nM_n\sum_{p=1}^{\infty}\min(\lambda_n\theta_p, \Delta_n+\delta_n^2)\to 0, \quad k_n\sum_{p=1}^{\infty}\min(M_n\lambda_n\theta_p,\Delta_n)\to 0,$$

*as $n\to\infty$ we obtain*

$$S_n \to_{n\to\infty} \mathcal{N}(0,\sigma^2), \quad \textit{in distribution.}$$

**Theorem 2** *Assume that the $E$-valued sequence satisfies the 'a'-weak dependence condition and the triangular array $(X_{n,k})_{1\le k\le k_n}$ defined as before satisfies assumption (6), then if*

$$(k_nM_n + k_n^{\frac{2}{3}})M_n\delta_n \to 0, \quad k_nM_n\sum_{p=1}^{\infty}\min(\lambda_n^2\theta_p, \Delta_n+\delta_n^2)\to 0, \quad \sum_{p=1}^{\infty}\min(\lambda_n^2\theta_p,\Delta_n)\to 0,$$

*as $n\to\infty$ we obtain*

$$S_n \to_{n\to\infty} \mathcal{N}(0,\sigma^2), \quad \textit{in distribution.}$$

D'autre part dans l'hypothèse (6), on peut remplacer $v_{k,n} = \operatorname{Var} S_{k,n} - \operatorname{Var} S_{k-1,n} \ge \alpha > 0$ par $\exists\, n_0\in\mathbb{N}^*/v_{k,n}>0\ \forall n\ge n_0,\ \forall 1\le k\le n$.

Il faut aussi remplacer la remarque "This result also yields the (standard) CLT $n^{-1/2}\sum_1^n\xi_k \longrightarrow \mathcal{N}(0,\sigma^2)$ for a stationary and bounded weakly dependent sequence with $\sigma^2>0$ and $\sum_{p=0}^{\infty}\theta_p<\infty$" de la page 65 par "A variation on the proof of the main result yields the standard CLT".

On remplace aussi 2.2.1. et 2.2.2. par

$s$-dependence:

Now $nM_n^2\delta_n + n^{\frac{2}{3}}M_n\delta_n = \frac{1}{\sqrt{nh}} + \frac{1}{n^{\frac{1}{3}}}$, so that we just need $nh\to_{n\to\infty}\infty$. Let $a\in]0,1]$,

$$nM_n\sum_{p=1}^{\infty}\min(\lambda_n\theta_p,\Delta_n+\delta_n^2) \le \sum_{p=1}^{\infty}\left(\frac{\theta_p}{h^2}\right)^a\left(\sqrt{\frac{h}{n}}\right)^{1-a} = \frac{1}{(nh)^{\frac{1-a}{2}}}h^{1-3a}\sum_{p=1}^{\infty}\theta_p^a.$$

We need that for some $a\le\frac{1}{3}$ , $\sum_{p=1}^{\infty}\theta_p^a<\infty$.

$$n\sum\min(\Delta_n, M_n\lambda_n\theta_p)\le\sum_{p=1}^{\infty}\min\left(h,\frac{n\theta_p}{nh^2}\right)\le\sum_{p=1}^{\infty}\theta_p^a h^{-2a}h^{1-a}.$$

If for some $0<a<\frac{1}{3}$, $\sum_{p=1}^{\infty}\theta_p^a<\infty$, the previous expression tends to 0 when $n\to\infty$.

$a$-dependence: Now, $nM_n^2\delta_n = \frac{1}{\sqrt{nh}}\to 0$ as $n\to\infty$. We also consider

$$nM_n\sum_{p=1}^{\infty}\min(\lambda_n^2\theta_p,\Delta_n+\delta_n^2) = \sum_{p=1}^{\infty}\min\left(\sqrt{\frac{h}{n}},\frac{\theta_p}{n^{\frac{1}{2}}h^{\frac{7}{2}}}\right)\le\frac{1}{\sqrt{nh}}h^{1-4a}\sum_{p=1}^{\infty}\theta_p^a \quad \text{for } a\in]0,1].$$

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So we need that exists $a \leq \frac{1}{4}$ with $\sum_{p=1}^{\infty} \theta_p^a < \infty$.

$$n \sum_{p=1}^{\infty} \min(\lambda_n^2 \theta_p, \Delta_n) = \sqrt{nh} \sum_{p=1}^{\infty} \min\left(\sqrt{\frac{h}{n}}, \frac{\theta_p}{n^{\frac{1}{2}} h^{\frac{7}{2}}}\right) \leq h^{1-4a} \sum_{p=1}^{\infty} \theta_p^a.$$

If there is some positive $a < \frac{1}{4}$ with $\sum_{p=1}^{\infty} \theta_p^a < \infty$, we conclude the proof.

# 2 Preuves des résultats principaux

On remplace, dans la preuve du Théorème 1, (11) page 67 par:

$$|\mathrm{Cov}(h^{"}_{k,n}(S_{k-1,n}), X^2_{n,k})| \leq \max(2C_2\ ,\ C_3) M_n \sum_{j=1}^{k-1} \min(\lambda_n \theta_j, \Delta_n + \delta_n^2). \tag{1}$$

On remplace (15) et (16) page 67 par:

$$|\Delta^{(1)}_{k,n}(h)| \leq \max(2C_2\ ,\ C_3) \left[ M_n^2 \delta_n + M_n \sum_{p=1}^{k-1} \min(\lambda_n \theta_p, \Delta_n + \delta_n^2) + \sum \min(M_n \lambda_n \theta_p, \Delta_n) \right] \tag{2}$$

et

$$\left| \sum_{k=2}^{k_n} \Delta^{(1)}_{k,n}(h) \right| \leq C \left[ k_n M_n^2 \delta_n + k_n M_n \sum_{p=1}^{\infty} \min(\lambda_n \theta_p, \Delta_n + \delta_n^2) + k_n \sum_{p=1}^{\infty} \min(M_n \lambda_n \theta_p, \Delta_n) \right]. \tag{3}$$

Dans la preuve du Théorème 2, on remplace

$$|\mathrm{Cov}(h''_{k,n}(S_{k-1,n}), X^2_{n,k})| \leq C M_n \sum_{j=1}^{k-1} \min(\lambda_n^2 \theta_j, \Delta_n),$$

par

$$|\mathrm{Cov}(h''_{k,n}(S_{k-1,n}), X^2_{n,k})| \leq C M_n \sum_{j=1}^{k-1} \min(\lambda_n^2 \theta_j, \Delta_n + \delta_n^2).$$

On remplace

$$|\Delta^{(1)}_{k,n}(h)| \leq C[M_n^2 \delta_n + M_n \sum_{p=1}^{k-1} \min(\lambda_n^2 \theta_p, \Delta_n) + \sum_{p=1}^{k-1} \min(\lambda_n^2 \theta_p, \Delta_n)]$$

par

$$|\Delta^{(1)}_{k,n}(h)| \leq C[M_n^2 \delta_n + M_n \sum_{p=1}^{k-1} \min(\lambda_n^2 \theta_p, \Delta_n + \delta_n^2) + \sum_{p=1}^{k-1} \min(\lambda_n^2 \theta_p, \Delta_n)].$$

Enfin on remplace

$$|\sum_{k=2}^{k_n} \Delta^{(1)}_{k,n}(h)| \leq C k_n M_n^2 \delta_n + k_n M_n \sum_{p=1}^{\infty} \min(\lambda_n^2 \theta_p, \Delta_n) + \sum_{p=1}^{\infty} \min(\lambda_n^2 \theta_p, \Delta_n)$$

par

$$|\sum_{k=2}^{k_n} \Delta^{(1)}_{k,n}(h)| \leq C k_n M_n^2 \delta_n + k_n M_n \sum_{p=1}^{\infty} \min(\lambda_n^2 \theta_p, \Delta_n + \delta_n^2) + \sum_{p=1}^{\infty} \min(\lambda_n^2 \theta_p, \Delta_n).$$