# Elementary partial differential equations: further suggested lectures and exercises

This note collects several references for suggested material lying in the natural continuation of the contents of the lectures, as well as suggested exercises to help you in your revisions. They should be considered as complementary or helpful resources for understanding the salient points of the course, and are by no means mandatory to take the exam.

**About the method of characteristics:** If you still feel confused with the method of characteristics, I suggest you read in detail the explanations in Section 12.2.2 in *[Haberman]*, and train yourselves with the subsequent exercises (a lot of them are corrected at the end of the book).

Then, you main go over again the numerous examples you have been proposed in the homeworks and midterms (see the corrections on my webpage). See also Exercises 1.2.1, 1.2.2, 1.2.3, 1.2.5, 1.2.6, 1.2.8, 1.2.9, 1.2.10 in *[Strauss]*.

**About establishing PDE from physical principles:** Sections 1.2 and 1.5, then 4.2 and 4.5 in *[Haberman]* are very detailed and worth reading. All the exercises of Section 1.2 are very good opportunities to test your understanding of the methods (see notably Exercises 1.2.1 and 1.2.9). See also exercises 1.5.1, 1.5.2 and 1.5.12 in the same reference.

To go further, you may also read Examples 5, 6 and 7 of Section 1.3 in *[Strauss]*.

**Generalities about the method of separation of variables:** Sections 2.4 and 2.5 in *[Haberman]* describe in details the resolution of a PDE with the method of separation of variables, and are worth reading if you do not feel so comfortable with it.

Exercises 4.1.2, 4.1.4, 4.1.5 and 4.2.2 in *[Strauss]* are very good opportunities to assess your understanding of the fundamentals of the method.

You may also see Exercises 4.1.6, 4.2.4 and 4.3.1, 4.3.2 in the same reference.

Eventually, Exercises 4.3.12 and 4.3.13 in *[Strauss]* are interesting and more challenging.

**About Fourier series:** You may appraise your ability in computing Fourier coefficients with Exercises 3.3.1, 3.3.2 and 3.3.3 in *[Haberman]*.

Exercises 5.4.4, 5.4.5 and 5.4.6 in *[Strauss]* are very easy, yet *crucial* revisions of even/odd functions in connection with Fourier series. See also Exercises 3.3.8 and 3.3.11 in *[Haberman]* in this context.

Exercises 5.3.3 and 5.3.5 in *[Strauss]* are (again) interesting uses of the method of separation of variables. Exercises 5.3.9 and 5.3.12 − 5.3.13 in *[Strauss]* are applications of the definition of symmetric boundary conditions. Exercise 5.3.10 in *[Strauss]* goes further and illustrates the Gram-Schmidt orthogonalization process for eigenfunctions, that we have only been evoking during the lectures.

Section 5.4 in *[Strauss]* contains a lot of worthwhile exercises to practice about the convergence theorems and the use of the $L^2$ theory for Fourier series. See notably exercises 5.4.1, 5.4.2, 5.4.4, 5.4.6, 5.4.9, 5.4.10, 5.4.12, and 5.4.13.

To practice the resolution of non homogeneous PDE by the method of separation of variables, see Exercises 5.6.4, 5.6.6, 5.6.7, 5.6.9 and 5.6.13 in *[Strauss]* (this last one is a little bit more difficult).

**Laplace equation and the method of separation of variables:** Exercises 6.2.1, 6.2.2 and 6.2.3 in *[Strauss]* are other interesting examples of handling the method of separation of variables (search for eigenvalues, dealing with boundary conditions, etc...) in the context of the Laplace equation.

In the same context, you may also see Exercises 6.4.1, 6.4.2, 6.4.6, 6.4.10 in *[Strauss]*.

Example 2 in Section 6.2 in *[Strauss]*, as well as Examples 2 and 3 in section 6.4. of *[Strauss]* are interesting illustrations of the method of separation of variables and Poisson's formula.